ESTIMATION OF CHARACTERISTICS OF DUTCH ROLL FOR MODEL AIRCRAFT AT THE CONCEPTUAL DESIGN PHASE

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ABSTRACT

The characteristics of aircrafts oscillatory modes are an important factor in an aircraft's performance. Dutch roll is one such important characteristic. The frequency, and the level of damping of dutch roll of an aircraft can be determined as of now. However, there are no methods to estimate it reliably at an earlier stage of design, when the exact details of the aircraft's design have not been determined. This paper investigates the estimation of the frequency and damping level of dutch roll on a subsonic aircraft weighing 1-5kg, using the values available and calculated at the conceptual design stage. These include the weight, wing and tail planforms and areas, and the approximate payload weight distribution. The objective of this study is to reduce the probability of an extensive redesign after the detail design, or in the case of most model aircraft, after the first flight.

Keywords: Dutch roll, Conceptual design, Subsonic flight, Lateral control

1. INTRODUCTION

An aircraft in flight can experience three oscillatory modes – phugoid mode, short period mode, and the dutch roll mode. Out of these, the dutch roll mode is the only oscillatory mode in the lateral plane [1].

The dutch roll is a coupled yaw-roll motion. It is usually initiated by an external disturbance (a gust, or a control input). When an aircraft has relatively low yaw stability relative to roll stability, a disturbance in roll produces a sideslip. The aircraft tends to correct its flight path using both roll and yaw. However, the roll component corrects the deviation faster, and the relatively weak yaw damping means the aircraft's yaw overshoots the desired position. Then, the opposing sideslip sets in, repeating the process. This motion is similar to that of a falling leaf.

The dutch roll mode is not a significant problem if it has a low frequency and/or if it is well damped, as the pilot can correct the tendency. However, violent dutch roll motion can be dangerous. For drones and unpiloted models, not having a pilot on board means that the mode may not be detected quickly enough in the worst case. The high dihedral in drones (used to control the spiral mode) aggravates the problem, as it increases roll stability.

This paper examines the methods of determining the frequency and damping of the dutch roll, as well as determining the stability of the motion. It also identifies the variables that must be identified and the quantities that must be approximated for these to be calculated.

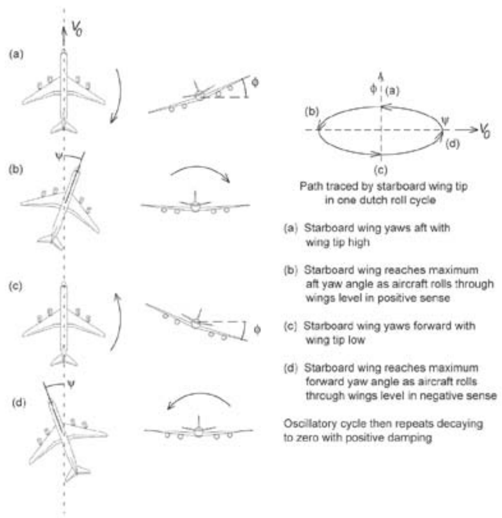

Figure 1: Dutch roll, from [2]

2. DETERMINATION OF FREQUENCY, DAMPING RATIO, AND STABILITY OF DUTCH ROLL MODE

Motion in the lateral-directional plane consists of roll, yaw, and sideslip. The transfer functions of these three parameters (relative to aileron input and rudder input) can be used to identify the roots. In the lateral plane, there are two exponential modes, the real roots which represent the roll mode and the spiral mode; and two oscillatory modes, the complex roots which represent two symmetric dutch roll modes [3].

The transfer functions can be obtained from linearized equations of small angle perturbations. From a longitudinal equilibrium state, this becomes [4, 5].

$$\left[\frac{d}{dt} - Y_v\right] v - Y_p p + [u_0 - Y_r] r - g_0 cos\Theta_0 \phi = Y_{\delta_r}\delta_r \qquad (01)$$

$$-L_v v + \left[\frac{d}{dt} - L_p\right] p - \left[\frac{I_{xz}}{I_x}\frac{d}{dt} + L_r\right] r = L_{\delta_r}\delta_r + L_{\delta_a}\delta_a \qquad (02)$$

$$-N_v v - \left[\frac{I_{xz}}{I_z}\frac{d}{dt} + N_p\right] p + \left[\frac{d}{dt} - N_r\right] r = N_{\delta_r}\delta_r + N_{\delta_a}\delta_a \qquad (03)$$

State variable vector: $x = [v\ p\ \phi\ r]^T$ (04)

Control vector: $\eta = [\delta_r\ \delta_a]^T$ (05)

This is equivalent to a system of 1st order equations which can be reduced to the standard form

$$\dot{x} = Ax + B\eta \qquad (06)$$

Where,

$$A = \begin{bmatrix} Y_v & Y_p & g_0 cos\Theta_0 & Y_r - u_0 \\ \frac{L_v + i_x N_v}{1 - i_x i_z} & \frac{L_p - i_x N_p}{1 - i_x i_z} & 0 & \frac{L_r + i_x N_r}{1 - i_x i_z} \\ 0 & 1 & 0 & 0 \\ \frac{N_v + i_z L_v}{1 - i_x i_z} & \frac{N_p + i_z L_p}{1 - i_x i_z} & 0 & \frac{N_r + i_x L_r}{1 - i_x i_z} \end{bmatrix} \qquad (07)$$

$$B = \begin{bmatrix} Y_{\delta_r} & 0 \\ \frac{L_{\delta_r} + i_x N_{\delta_r}}{1 - i_x i_z} & \frac{L_{\delta_a} + i_x N_{\delta_a}}{1 - i_x i_z} \\ 0 & 0 \\ \frac{N_{\delta_r} + i_z L_{\delta_r}}{1 - i_x i_z} & \frac{N_{\delta_a} + i_z L_{\delta_a}}{1 - i_x i_z} \end{bmatrix} \qquad (08)$$

Where $i_x = \frac{I_{xz}}{I_x}$ and $i_z = \frac{I_{xz}}{I_z}$.

For most aircraft, i_x and i_z are negligible, and therefore will be neglected for further analysis. This gives:

$$A = \begin{bmatrix} Y_v & Y_p & gcos\Theta & Y_r - u_0 \\ L_v & L_p & 0 & L_r \\ 0 & 1 & 0 & 0 \\ N_v & N_p & 0 & N_r \end{bmatrix} \qquad (09)$$

The transfer function is obtained by $|A - \lambda I|$:

$$\lambda^4 - \lambda^3(L_p + N_r + Y_v) + \lambda^2(Y_v L_p - L_v Y_p + Y_v N_r - N_v(Y_r - u_0) + L_p N_r - L_r N_p) + \lambda(-gcos\Theta . L_v - Y_v L_p N_r + Y_v L_v N_p + Y_p L_v N_r - Y_p L_r N_v - (Y_r - u_0) N_v L_p) + (-gcos\Theta L_{rN_v}) = 0 \qquad (10)$$

The roots of the transfer function can be determined using MATLAB. The equation has four roots:

- Two real roots λ_{roll} and λ_{spiral} representing the two exponential modes,
- Two complex roots $\lambda_D = a \pm bi$ which represent the dutch roll mode.

The frequency, damping, and stability of the dutch roll modes can be identified as follows [6].

Damping ratio:

$$\varsigma_{DR} = \sqrt{\frac{1}{1 + \left(\frac{b}{a}\right)_{DR}^2}} \qquad (11)$$

Undamped natural frequency:

$$\omega_{nDR} = \frac{-a}{\varsigma_{DR}} \qquad (12)$$

Where, ς_{DR} is the damping ratio, ω_{nDR} is the undamped natural frequency, a is the real part of the root, and b is the complex part. Stability can be determined from the sign of the real part of the root.

3. IDENTIFICATION OF VARIABLES AND LIMITS

The quantities that must be identified for the previous calculations are as follows:

g – specific gravity
Θ – angle between gravity and YZ plane of the

body axis system
u_0 – airspeed
Dimensional stability derivatives:
Y_v, Y_p, Y_r, L_v, L_p, L_r, N_v, N_p, N_r

These can be obtained from their respective aerodynamic derivates [7] ($C_{y\beta}$, C_{yp}, C_{yr}, $C_{l\beta}$, C_{lp}, C_{lr}, $C_{n\beta}$, C_{np}, C_{nr}) as shown in the table below [6].

Table 1: Determination of dimensional stability derivatives from aerodynamic derivatives in the lateral-directional plane

Variables	Y	L	N
v	$Y_v = \frac{QS}{mu_0} C_{y\beta}$	$L_v = \frac{QSb}{I_x u_0} C_{l\beta}$	$N_v = \frac{QSb}{I_z u_0} C_{n\beta}$
p	$Y_p = \frac{QSb}{2mu_0} C_{yp}$	$L_p = \frac{QSb^2}{2I_x u_0} C_{lp}$	$N_p = \frac{QSb^2}{2I_x u_0} C_{np}$
r	$Y_r = \frac{QSb}{2mu_0} C_{yr}$	$L_r = \frac{QSb^2}{2I_x u_0} C_{lr}$	$N_r = \frac{QSb^2}{2I_x u_0} C_{nr}$

Therefore, the following parameters must be determined from the model:

m – mass of aircraft
Q – dynamic pressure
S – wing area
b – wing span
I_x – moment of inertia, roll
I_z – moment of inertia, yaw
I_{xz} – product of inertia, roll and yaw

4. DETERMINATION OF VALUES OF VARIABLES AT CONCEPTUAL DESIGN LEVEL

In order to calculate the characteristics of the dutch roll mode at the conceptual design stage, the variables identified must be determined at the conceptual design stage. If this is not possible, an estimate must be substituted.

Out of the variables required,

- g is a constant,
- Θ can be considered 0 for straight and level flight.
- Several cases of u_0 must be selected for analysis (stall speed, designed cruise speed, designed top speed, or a speed in between). This is important as dutch roll is very dependent on the airspeed of the aircraft.
- The mass, m, is one of the basic parameters in the aircraft's design.
- Q can be determined from $Q = \frac{1}{2}\rho V^2$ using the speed that is to be used for calculation.
- Both S (wing area) and b (wing span) are determined very early in conceptual design.
- I_x, I_z, and I_{xz} can be approximated relatively easy by either
    - Modeling the aircraft using the approximate shape of its components (i.e., using boxes or plates to represent wings, for example) or
    - Using a CAD model to determine the moments of inertia

However, the determination of aerodynamic derivatives, which are dimensionless functions of control inputs and aircraft position, directly from the model is difficult. In particular,

- $C_{l\beta}$ is known as the dutch roll derivative, and is dependent on wing dihedral, wing sweep, geometry, fuselage geometry, and fins amongst other things [7]. This is one of the most important derivatives. Unfortunately, $C_{l\beta}$ and the corresponding stability derivative L_v are also two of the most difficult quantities to estimate.
- C_{lp} is the rolling moment due to roll rate and is a measure of roll damping.
- C_{lr} represents the same effect in yaw, but it's of a relatively low magnitude. L_r, the related stability derivative can be assumed to be $\left(\frac{1}{6}-\frac{1}{4}\right) \times C_L$ (C_L known) for a high aspect ratio rectangular wing [2].
- $C_{n\beta}$ determines the weathercock frequency and is dependent on the vertical tail. It determines the tendency of the aircraft to turn into the wind following sideslip [7].
- C_{nr} determines weathercock damping. N_r can be approximated as $-\frac{1}{2}\rho(u_0)_f S_f C_{L\alpha f} l_f^2$, which is less than 0. The values with subscript f indicate values relating to fin, l is the moment arm of the tailfin [2].
- C_{np} is the yawing that results from roll, a result of the differential drag on wings during a rolling maneuver. N_p can be approximated as $\frac{1}{2}\rho u_0^2 Sb\left(\frac{-C_L}{8}\right)$ for a rectangular wing [2].
- Out of $C_{y\beta}$, C_{yp}, C_{yr}, only $C_{y\beta}$, which is the sideslip effect of sideways motion, is significant. C_{yp} and C_{yr} can be assumed to be equal to zero [8].

The possible methods for extracting the four remaining derivatives (and more general values for the three approximated stability derivatives) include using a Taylor series expansion around a selected aircraft orientation, or using values obtained from similar aircraft.

The first approach is likely to be difficult and is not likely to be particularly more accurate than the latter option, given the small angle approximation and the fact that the small angle approximation is included in the derivation of the transfer function. The second approach can be used, but it requires an extensive amount of data and a system to determine the values that should be selected. I intend to compile this table as a second stage of this study.

5. PROPOSED FURTHER STUDIES

This study does not address the problem of identifying four of the nine aerodynamic derivatives. Either composing a table or determining a method for calculating them is essential for this method to be applied fully.

In addition, an iterative method that can be used with each iteration of the conceptual design process, (to determine its control characteristics each time its control surfaces are resized) would be very useful for the design process.

6. REFERENCES

[1] A. Deperrois, "*About stability analysis using XFLR5*", 2010 [online], At: www.xflr5.com/docs/XFLR5_and_Stability_analysis.pdf.

[2] "*Aircraft stability and control*", MIT Open courseware, 2004 [online], At: http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-333-aircraft-stability-and-control-fall-2004/, [Cited: 25th Feb 2015].

[3] H. B. Pettersson, "*Variable stability transfer function simulation*", MSc thesis, Virginia Polytechnic Institute & State University, 2002.

[4] D. Mc Lean, "*Automatic flight control systems*", S.l : Prentice Hall International, 1990.

[5] B. Stojiljkovic et al., "*The application of the root Locus method for the design of pitch controller of an F-104A aircraft*", s.l.: Journal of mechanical engineering, Vol 55, 2009.

[6] "*Dynamic stability*", Chapter 5, [Online], At: https://courses.cit.cornell.edu/mae5070/DynamicStability.pdf, [Cited: 20th Feb 2015].

[7] H. Loewen, "Stability derivatives", 2013 [Online], At: www.micropilot.com/pdf/stability-derivatives.pdf.

[8] B. B. Klawans, and J. A. White, "A method utilizing data on the spiral, roll-subsidence, and dutch roll modes for determining lateral stability derivatives from flight measurements", S.l : National advisory committee for aeronautics, 1957.