ESTIMATION OF NATURAL PERIODS OF A STRUCTURE

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ABSTRACT

Natural frequency is the frequency at which a system naturally vibrates once it has been set into motion. Natural frequency of a structure is used to predict the response of a structure under an extreme loading condition. If the dominant period of an external dynamic loading acting on a structure is equal to one of the natural periods of the structure, then the structural response will be amplified greatly due to the effect of resonance and hence, the structure may lead to a complete collapse. Therefore, to identify the dominant natural periods of a structure, ambient vibration measurements in time domain of the structure are converted into the frequency domain using the Fourier Transformation. Finally, this paper presents the identified dominant periods of two reinforced concrete frame buildings through the Fourier Transformation.

***Keywords*:** natural frequency, dynamic loading, dominant period

1. INTRODUCTION

The most important dynamic characteristic of a structure to be considered in designing for dynamic loading such as wind loadings and earthquake loadings is its natural periods [3]. If the dominant period of an external dynamic loading acting on a structure is equal to one of the natural periods of the structure, then the structural response will be amplified greatly due to the effect of resonance and hence, the structure may lead to a complete collapse. Therefore, it is important to identify the dominant natural periods of a structure.
Ambient vibration measurements are commonly used to identify the natural periods in the assessment and structural health monitoring of civil engineering structures such as buildings, bridges and towers because the ambient vibration testing is cheap, fast and no elaborate excitation equipment are required. Natural periods of a structure can be estimated using the Fourier Transformation technique. Furthermore, the extracted natural periods can also be used for verifying the design characteristics of a civil engineering structure and validating the numerical model that can be used to predict the response of a structure under an extreme loading condition.
This study is performed to identify dominant periods of two reinforced concrete frame structures through the ambient vibration measurements. In identifying the dominant natural periods, the time domain ambient vibration measurement is converted into the frequency domain response by using the Fourier Transformation technique.

2. FOURIER TRANSFORMATION

Fourier transformation [1], which is a linear transformation, transforms a given function *x(t)* in the time domain into the frequency domain using a basic function e^{-jwt}. This can be represented in the following form:

$$FT_{[x]} = \int_{-\alpha}^{\alpha} x(t) e^{-jwt} dt \qquad (01)$$

In the ambient vibration measurement of a structure, the signal is known only at N instants separated by the sampling interval. Therefore, Discrete Fourier Transform (DFT) is used to convert the time domain signal into frequency domain. Actually, DFT is equivalent to the continuous Fourier Transform for signal known only at N instants separated by sampling interval *T*. It can be expressed as in eq. (02).

$$X(k) = \sum_{n=0}^{N-1} x(k) e^{=j\frac{2\pi k}{N}n} \quad k = 0,1,..N-1 \qquad (02)$$

However, in this study, DFT is calculated by the numerically more efficient algorithm called Fast Fourier Transformation (FFT) algorithm [2]. The order of the computation for FFT is $N\log_2 N$ while it is for direct DFT is N^2.

3. BUILDING DESCRIPTION

The building 1 is a 3 storey reinforced concrete wall building which is shown in Figure 1(a). Initially, it was a reinforced concrete frame building but later, it has been retrofitted with lightly reinforced concrete walls. It could be considered to be symmetric in plan and elevation. The building has one bay of 6.7m width in the transverse direction and the 12 bays with equal width of 4.4m in the longitudinal direction. Altogether there are six reinforced concrete walls with equal cross section of 4.1x0.2m in the longitudinal direction. They are continuous to the roof with the same cross section. In-fill walls for interior partitioning are mainly in the transverse direction.

The building 2 is a 3 storey reinforced concrete frame building as shown in Figure 1(b). This building has a single bay of 7.49m width in the transverse direction while it has 10 bays with equal bay width of 4.22m in the longitudinal direction. This could also be considered to be symmetric in the plan and the elevation. It is important to note that there are series of short columns along the longitudinal direction and they are formed due to the presence of openings in the in-fill walls. This building was slightly damaged due an earthquake excitation and all the damages are concentrated in the short columns. The initiation of diagonal cracks in those short columns can be observed. However, there is no other damage observed in in-fill walls either in the longitudinal or transverse directions.

Ambient excitations are measured at each floor level of the building and the ground during the field test by tri-axial seismometers (CMG-6TD). They are placed approximately at the centre of each floor plan. All data is acquired at the frequency of 100Hz for the period of 1800 seconds. The recorded data is then pre-processed using a band pass filter (Chebyshev) at the specified frequency range from 2 to 15Hz to remove the non-zero mean noise and the uncorrelated to the structural response.

(a) (b)

Figure 1(a) Building 1 and (b) Building 2

4. RESULTS

Figure 2(a) and 2(b) illustrate the filtered acceleration time history at the top story level of the building 1 in East-West (E-W) direction and their Fourier transformation (FT) plot, respectively while Figure 3(a) and 3(b) shows the acceleration time history at the top story of the same building in North-South (N-S) direction and its FT plot, respectively.

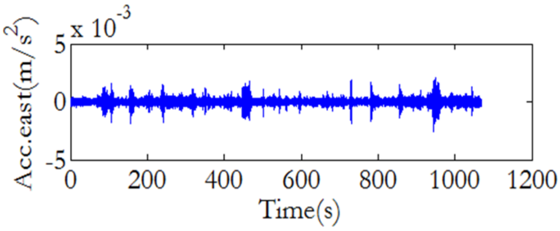

Figure 2(a): E-W component of acceleration time history at top storey in building 1

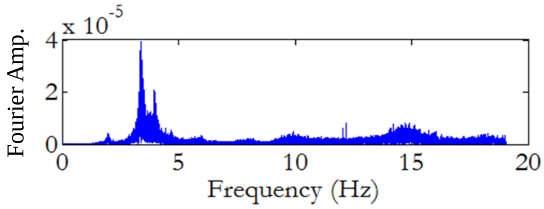

Figure 2(b): Fourier spectrum of E-W component of acceleration- building 1

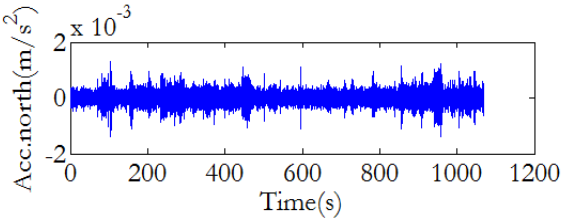

Figure 3(a): N-S component of acceleration time history at top storey in building 1

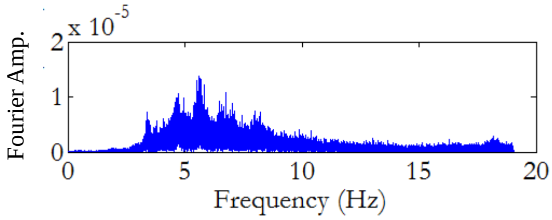

Figure 3(b): Fourier spectrum of N-S component of acceleration- building 1

Figure 4(a) and 4(b) illustrate the filtered acceleration time history at the top story level of the building 1 in East-West (E-W) direction and their Fourier transformation (FT) plot, respectively while Figure 5(a) and 5(b) shows the acceleration time history at the top story of the same building in North-South (N-S) direction and its FT plot, respectively.

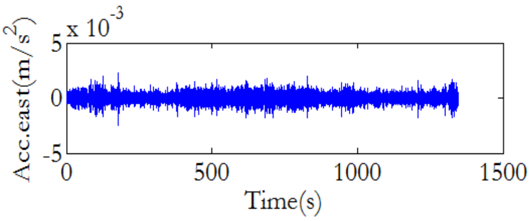

Figure 4(a): E-W component of acceleration time history at top storey in building 2

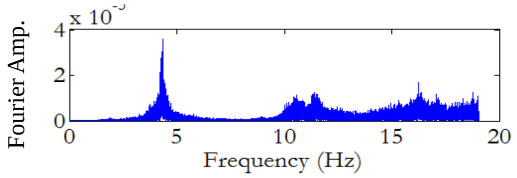

Figure 4(b): Fourier spectrum of E-W component of acceleration- building 2

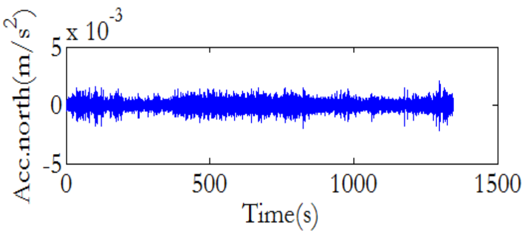

Figure 5(a): N-S component of acceleration time history at top storey in building 2

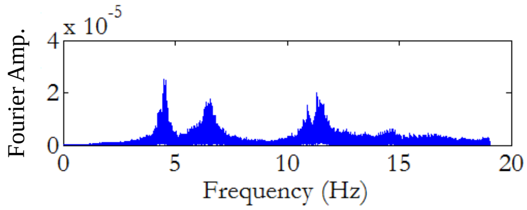

Figure 5(b): N-S component of frequency time history at top storey in building 2

Analysed results by average method –

Figure 6(a) and 6(b) illustrate the Fourier transformation (FT) plots of measured response of 1200 seconds and the average response of 300 seconds, respectively while Figure 7(a) and 7(b) shows the Fourier transformation (FT) plots of measured response of 1200 seconds and the average response of 300 seconds, respectively.

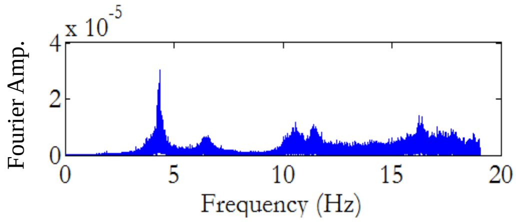

Figure 6(a): E-W component of frequency time history at top storey in building 2 (1200 s)

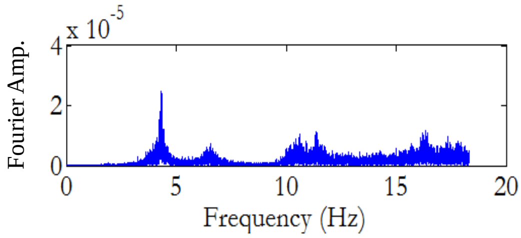

Figure 6(b): E-W component of frequency time history at top storey in building 2 (300 s)

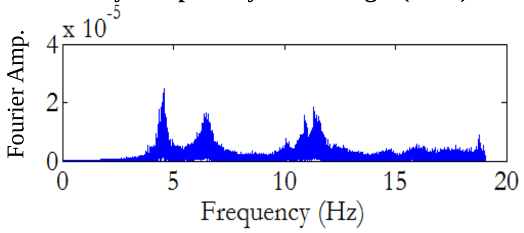

Figure 7(a): N-S component of frequency time history at top storey in building 2 (1200 s)

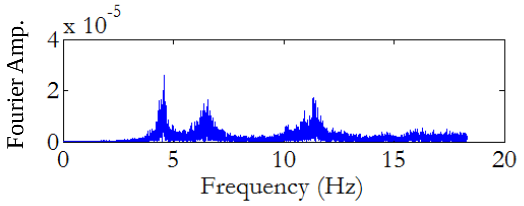

Figure 7(b): N-S component of frequency time history at top storey in building 2 (300 s)

Table 1 shows the natural frequencies of building 1 identified in the Fourier Transformation plots shown in Figure 2 and 3. Table 2 shows the dominant natural frequencies of building 2 obtained in the Fourier Transformation plots shown in Figure 4 and 5. It should be noted that four natural modes of vibration can be identified.

Table 1: Identified Frequencies of Building 1

Mode	Frequency (Hz) (E-W)	Frequency (Hz) (N-W)
1	3.367	3.401
2	3.997	4.714
3	9.174	5.586
4	14.924	7.143
5	-	8.197

Table 2: Identified Frequencies of building 2

Mode	Frequency (Hz) (E-W)	Frequency (Hz)/ (N-W)
1	4.651	4.525
2	10.638	6.578
3	11.494	10.625
4	16.393	11.500

5. CONCLUSION

By Fourier Transformation, time domain signals can be converted into signals in the frequency domain. Fourier transformation can be used to identify dominant periods of structures. The study indicated that the highest period of building 1 is 0.299s and the highest period of building 2 is 0.294s.

Noise frequencies caused by disturbances can be eliminated by the average method. The average method is useful to smooth the graphs and thus the dominant frequencies can be clearly identified.

6. REFERENCES

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