KINEMATICS AND DYNAMIC MODELING OF A SURFACE VESSEL FOR MODEL BASED CONTROLLING

K.J.C. Kumara

Doctoral Candidate, Graduate School of Asia Pacific Studies, Ritsumeikan Asia Pacific University, Japan.
Email: kumara.kjc@ieee.org

ABSTRACT

This paper presents kinematics and dynamics analysis of a general model of an Autonomous surface vessel (ASV) in the ocean. The novel vehicle (Fig.1.) mainly consists of two hulls (pontoons), a strut of hydrofoil cross section, a submerged body (Gertler body) connected at the bottom end of the strut and two propellers. The vessel moves in a hydrodynamic environment where many uncertainties, non-linear and non-predictive behaviors always appear. The ocean vehicle is modeled mathematically using first principles and derivations wherever possible. This type of practically close model is crucial for researchers who develop controllers for surface vessels to simulate and for further tuning as prior testing of a physical model is quite expensive.

***Key words*:** Autonomous surface vessel, Vessel Control, Vessel Dyanmics.

1. INTRODUCTION

Research and development of classical controllers for any dynamical system are initially based on the availability of a mathematical model, which provides close and accurate description of dynamics of the real system. Analysis of dynamics of an ocean surface vehicle is motivated by the need to provide more field-oriented and mathematical model for standard vessel as then it will be even used to compare performances of various controllers.

Many previous research publications are available on mathematical modeling, control, navigation, and communication systems for underwater vehicles [1]--[4], but only a limited number of research has been reported on surface vessels, especially for autonomous applications [4], [5], [6] with engine propulsions. A mathematical model for a hovercraft was developed in [7] to introduce stabilization algorithms. This problem is simple when compared to that of a standard surface vessel as it does not contain many of the non-linear coupled drag terms. In [7] and [8] also, mathematical models were used to test and simulate control algorithms but these rather poorly represent the non-linear dynamics of the vehicle.

Most of the researchers have not addressed the following important issues when developing dynamics relations for their models and some of them are only partially analyzed [4], [6], [7].

- Angular speed and torque of propellers' actions (or propellers) which powered the motion and control the vessel by contributing as force and moment inputs (thrust).
- Unsteady forces induced on the accelerating vessel and where all the relations developed to find resistance forces are failed.
- Validity of assumption that the residual resistance is independent of the Reynolds number (R_n), but this is not accurate and failed in many cases.

But the aforementioned issues are very much important to be addressed [2] to realize an accurate model for an ASV because the ocean environment where it operates is characterized by unknowns, undesirables, and perturbations. In this paper, these issues are studied and possible solutions are given using theories described in [2] and further forces and moments calculated using practical data found in [2], [4], [4],[11] and references there in.

2. MAIN COMPONENTS OF THE SURFACE VESSEL

A geometric model of the ASV (Fig.1.) is generated for the dimensions depicted in Table 1 by using Pro/E CAD package [19]. The design of an ASV depends on many factors and conditions.

But here, geometry of the vehicle is important only as mathematical relations are obtained using general notations rather than numerical values. Fluid properties are assumed to be as in Table 2 for temperature 10^0C and 3.3% salinity [2]. The planform area depends heavily on fluid density. The fluid properties are particularly required in friction calculations described in section IV.

A. Hulls of the Vessel

Two hulls (pontoons) with same shape and dimensions are mounted symmetrically on either side of the body. Since beam to length ratio (B_h / L_h) is small, hulls are approximated to flat plates for damping calculations.

B. Strut

Strut is a thin column with symmetric airfoil cross section extended from bottom of the body to inside of the sea surface. An in-depth discussion of airfoil sections and design can be found in [10].

C. Gertler Body

Submerged body [9] is another name used to describe this pontoon which is fixed to bottom end of the strut, is mainly for stabilization purpose of the vessel.

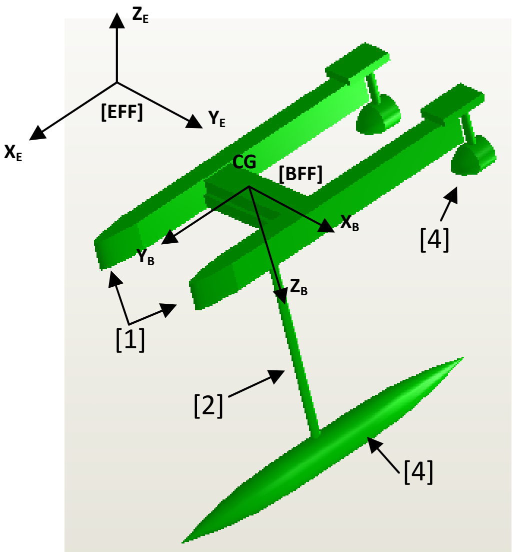

Fig.1: design of the ASV (where [1] hulls, [2] vertical strut, [4] Gertler body, and [4] propellers) and Coordinate frames

D. Propellers

Two propellers are mounted at the back end of the hulls by vertical supports and the thrust inputs are generated by the propulsive action of the power drives. Please refer [11] for different types of propellers and related issues.

3. COORDINATE FRAMES AND KINEMATICS

The nomenclature defined in [1] is used throughout this paper.

A. Coordinate Frames

Fig 2 shows two reference frames. The earth fixed frame (EFF) is chosen so that the centre of gravity (CG) of the vessel is at the origin at time, t = 0. The body fixed frame (BFF) is chosen such that its origin is fixed at the CG of the vessel as shown in Fig 2.

Three degrees-of-freedom (d.o.f.) system is considered by neglecting vertical (heave), pitch, and roll motions [4], [6], [7], [5]. It is assumed that this popular assumption is valid without appreciable loss in accuracy under typical and slightly severe maneuvers.

The configuration vector of the ASV that is the vector of BFF w.r.t. EFF is given as

$$\eta(t) = [x, y, \psi], \quad t \geq 0 \qquad (1)$$

where $x = x(t)$, $y = y(t)$ represent the linear displacement and $\psi = \psi(t)$ is the rotation about the vertical Z_E-axis.

Table 1

Body dimensions of the asv

Component	Parameter	Notation	Value
Hulls	Length	L_h	2.54 m
	Beam	B_h	0.1524 m
	Draft	T_h	0.1270 m
	Planform area	A_h	0.7742 m^2
Vertical strut	Length	L_s	1.58 m
	Beam	b_s	0.025 m

Component	Parameter	Notation	Value
	Chord	c_s	0.075 m
	Projected area (X_B direction)	S_x	$0.035 m^2$
	Projected area (Y_B direction)	S_y	$0.1185 m^2$
Gertler body (series model 4154 in [9] is used)	Length	L_g	1.87 m
	Greatest diameter	d_g	0.443 m
Propellers (screw type)	Blade diameter	d	0.1 m
	No of blades	n_p	3

Table 2

Fluid properties

Property	Notation	Value
density	ρ	$1025\ kg/m^3$
viscosity	υ	$1.4\times10^{-6}\ m^s/s$

B. Kinematics

By defining velocities of surge (X_B-direction), sway (Y_B-direction), and yaw (rotation about Z_B-direction) as $u = u(t)$, $v = v(t)$, and $r = r(t)$, then the velocity vector of the ASV can be written as

$$V(t) = [u, v, r],\ t \geq 0. \tag{2}$$

When the angle of trim (θ) and angle of roll (γ) are negligible the rotation matrix from the BFF to the EFF for a 3 d.o.f. system can be derived to be [1]

$$J(\eta) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Then, the relation between the $\eta(t)$ and $V(t)$ can be expressed as

$$\frac{d(\eta(t))}{dt} = \dot{\eta}(t) = J(\eta)V(t). \tag{4}$$

C. Equations of Motion

Newton-Euler equations governing the motion of the ASV under the influence of external forces and torques are written referring to Fig. 1. The resulting general equations of motion of the ASV along the directions of BFF are

Surge : $m(\dot{u} - vr - y_G\dot{r} - x_G r^2) = X$

Sway : $m(\dot{v} - ur + x_G\dot{r} - y_G r^2) = Y$

(5)

Yaw :

$I_Z\dot{r} + m[x_G(\dot{v} + ur) - y_G(\dot{u} - vr)] = N$

where X , Y , and N are the external forces and moment acting on the vehicle. x_G and y_G are the distances to the CG of the ASV from the origin of the BFF. Here, by placing origin of the BFF at the CG, x_G, $y_G \to 0$, and the above equations are simplified. The above equations of motion, (5), are rewritten as

$$M\dot{V}(t) + [C(v) + D(v)]V(t) + g[\eta(t)] = T_R(t);\ t \geq 0 \tag{6}$$

where M is the positive definite mass-inertia matrix, $C[v(t)] \in \Re^{3\times3}$ is the total matrix of Coriolis and centripetal terms, $D[v(t)] \in \Re^3$ is the damping vector, $g[\eta(t)] \in \Re^3$ is the vector of gravitational forces and moments, and $T_R(t) = [X(t), Y(t), N(t)]^T \in \Re^3$ is the input vector that represents the external forces and moments that will be described shortly.

Further, M and $C[v(t)]$ consist of two components representing rigid body (RB) and added mass (AM) matrices as follows [2]

$$M = M_{RB} + M_{AM} \tag{7}$$

$$C = C_{RB} + C_{AM} \tag{8}$$

where

$$M_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_Z \end{bmatrix} \quad \text{and}$$

$$C_{RB} = \begin{bmatrix} 0 & -mr & 0 \\ mr & o & o \\ 0 & 0 & 0 \end{bmatrix}.$$

Added mass components, M_{AM} and C_{AM}, are due to the forces of the surrounding fluid and are further described by (49) and (50) in Section IV of this paper.

The gravitational forces and moments include the forces and moments due to the weight (W) of the vessel and the buoyancy (b) force

$$g[\eta(t)] = g_W(t) + g_b(t) \tag{9}$$

where $g_W = mg$ and $g_b = -\rho(volume)g$. As vertical (heave) motion is neglected, $g[\eta(t)] = 0$.

4. DYNAMICS OF THE ASV

As the ASV dynamics depend on external hydrodynamic damping forces, it becomes essential to analyze all resistant forces; drag, viscous friction, residual or wave friction, and other ways of friction forces. Afterwards, a complete damping matrix will be derived and hence the mathematical model.

a. Analysis of Hydrodynamics Damping Forces and Moments

The velocity of any point of the vehicle can be represented using linear and angular velocity couplings derived from velocities at CG and distances, d_x, d_y (in X_B and Y_B-directions), from the CG to the point considered. To this end, the velocity of any point can be written as

$$\begin{aligned} u_{pt} &= u(t) - d_y r(t) \\ v_{pt} &= v(t) + d_x r(t) \\ r_{pt} &= r(t). \end{aligned} \tag{10}$$

Also, total resistance on the ASV can be split up into [11]

1. frictional resistance or viscous friction (R_{fric})
2. form resistance (R_{form}) and
3. wave (residual) resistance (R_{res})

2 and 3 together constitute Froude's residual resistance. The total resistant coefficient can then be written as

$$C_t = (1+k).C_f + C_w + C_a \tag{11}$$

Where

C_f - frictional resistance coefficient

C_w - wave resistance coefficient

C_a - additional resistance coefficient and

$(1+k)$ - form factor where k is a constant.

From Froude's approach [11]

$$C_f = \frac{R_{fric}}{\frac{1}{2}\rho V^2 S} \tag{12}$$

where

ρ – density of the sea water in kg/m^3

V – model speed

S – wetted surface area of the object/hull in m^2.

Several relations have been developed in [2] and [11] to calculate C_f based on Renold number, R_n, defined as

$$R_n = \frac{Ul}{\upsilon}. \quad (13)$$

And, R_n is used to estimate resistance coefficients [4] with correct substitution for characteristics velocity, U, which is defined same as u_{pt} in (11) and characteristics length, l. ITTC–1957 (International Towing Tank Conference) accepted (14) as the accurate relation [2]. Therefore, (14) is used in this discussion as well.

$$C_f = \frac{0.075}{[\log(R_n) - 2]^2}. \quad (14)$$

Wave resistance coefficient is [11]

$$C_w = C_t - (1+k)C_f \quad (15)$$

where $(1+k)$ is known as form factor depicted in Table 3 for different block coefficients (C_B). The block coefficient is the ratio between the ship's displacement volume and that of a rectangular box in which the ship's underwater volume "just fit".

Table 3

The form factor

C_B	$1+k$
≤ 0.7	1.10 ~ 1.15
0.7 ~ 0.8	1.15 ~ 1.20
≥ 0.8	1.20 ~ 1.30

A simple formula was defined by Holtop & Mennen, 1982, and described in [11] is used to estimate C_a using water plane length, LWL, as

$$C_a = 0.006.(LWL + 100) - 0.00205. \quad (16)$$

Then, the total resistance force (in magnitude and direction) can be generalized as

$$F_t = \frac{1}{2}\rho AU \; |U| C_t \quad (17)$$

where A is projected/effective area and U is the velocity at CG of the particular object or element calculated same as u_{pt} in (10).

b. Damping Forces on Hulls

Resultant force in X_B-direction is contributed by viscous/skin drag (VD) and residual resistance /wave drag (WD) for both hulls (port side hull and starboard side hull) are given by [2]

$$F_{h,X_B}^{VD}(u,r) = C_f \rho A_h U|U| \quad (18)$$

$$F_{h,X_B}^{WD}(u,r) = C_W \rho A_h U|U| \quad (19)$$

where $U = u - d_{h,y} r$ and $d_{h,y}$ is the distance from the CG to the port side hull (same for starboard side hull as well) in Y_B-direction and, A_h is the wetted planform area of the either hull.

Damping force in Y_B-direction can be calculated by integrating elemental damping forces for both hulls as

$$F_{h,Y_B}^{D}(v,r) = \int_{aft}^{forward} dFy = \int_{-dax}^{dfx} \frac{1}{2} C_{Dh} T_h \rho v(x)|v(x)| dx$$

(20)

where

$$v(x) = v + \frac{v_{bow} - v_{stern}}{L_h} \quad (21)$$

with v_{bow} and v_{stern} being the velocities at bow (front end) and stern (back end) of the hulls, and $C_{Dh} = 2$ for cross flow over a flat plate [4].

Total damping moment in ψ_B direction is calculated by summing the total moments generated by the forces in X_B and Y_B- directions as

$$m_h(u,v,r) = m_{h,x}(u,r) + m_{hp,y}(v,r) + m_{hs,y}(v,r) \quad (22)$$

where $m_{h,x}(u,r)$ the moments exerted by the forces in the X_B -direction and

$$m_{hp,y}(v,r) = m_{hs,y}(v,r) = \int_{aft}^{forward} x.dF_y \; .$$

c. Strut Damping Forces

Damping forces on the strut consist of both viscous drag and wave drag and the prior one is derived based on lift (L) and drag (D) forces exerted on the hydrofoil section. In this work, we adopt the hydrofoil section of the type NACA 0012 described in [10]. The angle of attack, α , which is defined as the angle between "nose-tail line" of the hydrofoil and free-stream direction is used to find the drag and lift coefficients (C_D and C_L). In [4], C_D and C_L were plotted for full range of α (i.e., $-180^0 < \alpha < 180^0$) and the same results are used in our work. To that end,

$$L = -\frac{1}{2}\rho U|U| S_y C_L \quad (23)$$

$$D = \frac{1}{2}\rho U|U| S_x C_D \quad (24)$$

where S_x and S_y are projected area of the strut in X_B and Y_B-directions respectively and U is defined same as u_{pt} in (10).

Therefore, the total force in the X_B -direction is

$$F^{VD}_{S,X_B}(u,r) = L\sin\alpha + D\cos\alpha \quad (25)$$

$$F^{WD}_{S,X_B}(u,r) = \frac{1}{2}\rho V_{S,X_B}\left|V_{S,X_B}\right| A_x C_{rx} \quad (26)$$

and that in the Y_B -direction is

$$F^{VD}_{S,Y_B}(v,r) = L\cos\alpha - D\sin\alpha \quad (27)$$

$$F^{WD}_{S,Y_B}(u,r) = \frac{1}{2}\rho V_{S,Y_B}\left|V_{S,Y_B}\right| A_y C_{ry} \quad (28)$$

where A_x and A_y are projected areas, V_{S,X_B} and V_{S,Y_B} are the velocities of the strut of the format as defined in (11). C_{rx} and C_{ry} are residual friction coefficients, respectively, in X_B and Y_B-directions, estimated based on Froude number, F_n , which is defined in (29) and empirical data in [4].

$$F_n = \frac{U}{\sqrt{gl}} \quad (29)$$

The total damping moment, $m_S(u,v,r)$, for the strut can then be derived by multiplying total forces in (25), (26), (27), and (28) by respective distances.

d. Gertler Pontoon Damping Forces

X_B -direction Gertler body force can be calculated as normal damping force in (17) using resistant coefficient (C_g) values interpolated for different R_n s as defined in (13) as follows

$$F^{VD}_{G,X_B}(u) = \frac{1}{2}\rho u|u| A_{g,x} C_g \; . \quad (30)$$

Total drag force in Y_B-direction is calculated by summing up elemental drag (F^{VD}_{G,Y_Bi}) as in (32). Pontoon is divided into 10 (i =1, …, 10) elements in Y_B-direction as cross section is different in each.

$$F^{VD}_{G,Y_Bi} = \frac{1}{2}\rho A_{yi} v_{gi}\left|v_{gi}\right| C_{gi} \quad (31)$$

$$F^{VD}_{G,Y_B} = \sum_{i=1}^{10} F^{VD}_{G,Y_Bi} \quad (32)$$

Damping moment in ψ_B-direction can then be derived as

$$m_G(u,v,r) = \sum F^{VD}_{G,Y_Bi}(v,r) d_{gi} \quad (33)$$

where $v_{gi} = v + d_{gi} r$, and d_{gi} is the distance from the CG of the pontoon to element CG in Y_B-direction. Moment derived from the force in (30) is cancelled as the vessel CG vertical plane is aligned with submerged body mid-vertical plane.

e. Thrust and Propulsion

A propulsion system is needed to overcome the resistance and drive the vessel in desired trajectories by desired velocities and accelerations. The basic action of propulsors (eg: propellers) is to deliver a thrust. Different propulsion configurations, specific applications and advantages can be found in [11]. The efficiency of propulsors varies widely depending on propulsion method used (type) and among above types, screw propellers has not yet been equaled in most cases for other types (greater efficiency) and is therefore the most commonly used type in ships. Two screw propellers are chosen for the model used in this work and their properties are depicted in Table 1. Sketch of a screw propeller is given in Fig. 3.

In the sequel, we use the following notations

T – Axial thrust force, Q – Shaft torque

U_a – Axial velocity, n - Angular velocity of blades

d - Diameter of the propeller.

Then, non-dimensional advanced ratio can be defined as [2]

$$J = \frac{U_a}{nd}. \tag{34}$$

Thrust, *T*, and torque, *Q*, can be derived using the thrust coefficient, K_T, and torque coefficient, K_Q, as follows [2], [11]

$$T = K_T \rho d^4 n^2 \tag{35}$$

$$Q = K_Q \rho d^5 n^2. \tag{36}$$

K_T and K_Q values for various advanced ratios can be found in [2] (page 26). Resultant thrust vector (input force and moment) is given by

$$T_R = [X \quad Y \quad N]. \tag{37}$$

When T_P and T_S are thrusts delivered by port side and starboard side propellers, respectively, α_P and α_S are inclinations to the X_B-axis, and d_{P,X_B} and d_{P,Y_B} respectively are distances in X_B and Y_B-directions, then

$$X = T_P \cos\alpha_P + T_S \cos\alpha_S \tag{38}$$

$$Y = T_P \sin\alpha_P - T_S \sin\alpha_S \tag{39}$$

$$N = X \times d_{p,Y_B} + Y \times d_{P,X_B}. \tag{40}$$

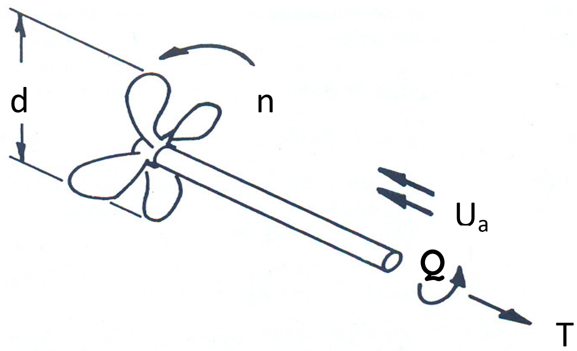

Fig. 2. Sketch of a Propeller and It's Shaft

f. State-space Representation

Here, we develop the state-space representation of the plant that is important in state-space methods of controller design and analysis.

Equation (6) can be rewritten as

$$\dot{V}(t) = -M^{-1}[C(v) + D(v)]V(t) + M^{-1}T_R. \tag{51}$$

Now, define the state vector as

$$X = [\eta^T \quad V^T]^T. \tag{52}$$

From (4) and (51), the state space model of the ASV can be obtained as

$$\dot{X} = f(X) + BT_R \qquad (53)$$

where $$B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \qquad (54)$$

and

$$f(X) = \begin{bmatrix} J(\eta)\, V(t) \\ -M^{-1}[C(v) + D(v)]V(t) \end{bmatrix} \qquad (55)$$

V. CONCLUSION

In this research, the focus has been primarily on the overall dynamics and control aspects of developing an autonomous surface vessel. This work is split into several steps. First, a geometric model of the surface vessel is generated by using a cad software. Second, an accurate and reliable mathematical model is developed by analyzing kinematics, vehicle body properties, damping forces and hydrodynamics behavior of the model. Third, two screw type propellers are suggested for the vessel and thrust inputs are analyzed and discussed important aspects of propellers separately. Finally, a Computed toque-like controller is designed and implemented in matlab environment for the mathematical model of the surface vessel.

APPENDIX

The following mild assumptions are made in the mathematical modeling of the ASV:

1. Surface vessel is rigid.
2. Earth fixed frame [EFF] is inertial.
3. Vehicle velocities in vertical, pitch and roll directions are insignificance
4. Gravity and buoyancy forces are acting on Z_E direction only and Z_B **is** always parallel to the Z_E.
5. Buoyancy forces are compensated by gravitational forces and then vector of gravitational forces are neglected.
6. Hulls are approximated as flat plates.
7. Fluid interference between the port and starboard hulls is negligible.
8. Thrusts are delivered by two propellers are same in magnitudes.

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